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$W^+ - H^+$ Interference and Partial Width Asymmetry in Top and Antitop Decays

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Abstract

We re-examine the question of a possible difference in the partial decay widths of t and $\bar{t}$, induced by an intermediate scalar boson H^+ with CP-violating couplings. The interference of W^+ and H^+ exchanges is analysed by constructing the 2×2 propagator matrix of the $W^+ - H^+$ system, and determining its absorptive part in terms of fermion loops. Results are obtained for the partial rate difference in the channels $t \to b l^+ \nu_l$ and $t \to b c \bar{s}$, which fulfil explicitly the constraints of CPT invariance. These results are contrasted with those in previous work.

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1 Introduction

Recent literature [1, 2] has been witness to an interesting debate on the question of a possible CP-violating difference in the partial widths of t and $\bar{t}$ decays, into conjugate channels such as $t \to b\tau^+\nu_\tau$ and $\bar{t} \to \bar{b}\tau^-\bar{\nu}_\tau$. This discussion has taken place in the context of a model in which the decays of the top quark are mediated, not only by $W^\pm$ bosons, but also by charged Higgs bosons $H^\pm$ with CP-violating couplings [3]. Two specific questions that have arisen in this regard are (i) the correct form of the propagator for an unstable W boson [1, 2, 4, 5], and (ii) the implications of CPT invariance and unitarity for partial rate asymmetries generated by absorptive parts of decay amplitudes [6].

In this paper, we present an analysis that, we believe, is more complete than that in Refs. [1, 2]. Central to our analysis is the derivation of the propagator matrix of the coupled $W^+ - H^+$ system, taking account of vacuum polarization effects induced by fermion loops. The propagator matrix includes off-diagonal transitions between W^+ and H^+, which turn out to be essential for obtaining a partial rate asymmetry that respects the constraints of CPT invariance.

The model we use is defined by the Lagrangian [3]

$$\mathcal{L} = \mathcal{L}_W + \mathcal{L}_H, \tag{1}$$

$$\begin{aligned}\mathcal{L}_W = & -\frac{g}{2\sqrt{2}}\Big\{\sum_{l=e,\mu,\tau}\Big[\bar{\nu}_l\gamma^\mu(1-\gamma_5)lW^+_\mu + \bar{l}\gamma^\mu(1-\gamma_5)\nu_l W^-_\mu\Big] \\ & +\bar{u}\gamma^\mu(1-\gamma_5)dW^+_\mu + \bar{d}\gamma^\mu(1-\gamma_5)uW^-_\mu \\ & +(u,d)\to(c,s)+(u,d)\to(t,b)\Big\},\end{aligned} \tag{2}$$

$$\begin{aligned}\mathcal{L}_H = & \frac{g}{2\sqrt{2}}\Big\{-\sum_{l=e,\mu,\tau}\Big[H^+Z\frac{m_l}{m_W}\bar{\nu}_l(1+\gamma_5)l + H^-Z^*\frac{m_l}{m_W}\bar{l}(1-\gamma_5)\nu_l\Big] \\ & +H^+\bar{u}\Big[Y\frac{m_u}{m_W}(1-\gamma_5) + X\frac{m_d}{m_W}(1+\gamma_5)\Big]d\end{aligned}$$

$$
\begin{aligned}
&+H^-\overline{d}\Big[X^*\frac{m_d}{m_W}(1-\gamma_5)+Y^*\frac{m_u}{m_W}(1+\gamma_5)\Big]u \\
&+(u,d)\to(c,s)+(u,d)\to(t,b)\Big\}, \qquad (3)
\end{aligned}
$$

where we neglect quark-mixing. The parameters X, Y, Z appearing in $\mathcal{L}_H$ are permitted to be complex relative to one another, so that this term is CP-violating. The interaction $\mathcal{L}_H$ may be imagined to arise as a special case of the Weinberg model with three Higgs doublets [7], in which the remaining charged scalars are sufficiently heavy to be disregarded.

2 The $W^+ - H^+$ Propagator

We are concerned with the propagator (in unitary gauge) of the coupled $W^+ - H^+$ system, which we describe by a 2×2 matrix

$$
D = \begin{pmatrix} D_W^{\mu\nu} & D^{\mu}_{W^+H^+} \\ D^{\nu}_{H^+W^+} & D_H \end{pmatrix}. \qquad (4)
$$

The inverse of this matrix is defined by

$$
D^{-1}D = \begin{pmatrix} g & 0 \\ 0 & 1 \end{pmatrix}, \qquad (5)
$$

where g is the metric tensor with elements $g^{\mu\nu} = diag(1,-1,-1,-1)$. The inverse matrix D^{-1} has the general form

$$
D^{-1} = -i\begin{pmatrix} (m_W^2 - q^2 + F_1)g^{\mu\nu} + q^\mu q^\nu (1+F_2) & q^\mu F_3 \\ q^\nu F_4 & q^2 - m_H^2 + F_5 \end{pmatrix}, \qquad (6)
$$

where the functions $F_i(q^2)$, $i = 1,\cdots,5$ are given by the one-particle-irreducible self-energies

$$
\Sigma_W^{\mu\nu}(q^2) \;=\; i\Big[g^{\mu\nu}F_1(q^2) + q^\mu q^\nu F_2(q^2)\Big],
$$

$$
\begin{aligned}
\Sigma^{\mu}_{W^+H^+}(q^2) &= iq^\mu F_3(q^2), \\
\Sigma^{\mu}_{H^+W^+}(q^2) &= iq^\mu F_4(q^2), \\
\Sigma_H(q^2) &= iF_5(q^2).
\end{aligned} \tag{7}
$$

Inversion of the matrix (6) yields the elements of the propagator matrix (4):

$$
\begin{aligned}
D_W^{\mu\nu} &= i\frac{-g^{\mu\nu}+q^\mu q^\nu \dfrac{(1+F_2)(q^2-m_H^2+F_5)-F_3F_4}{(m_W^2+F_1+q^2F_2)(q^2-m_H^2+F_5)-q^2F_3F_4}}{q^2-m_W^2-F_1}, \\
D^{\mu}_{W^+H^+} &= \frac{iq^\mu F_3}{q^2F_3F_4-(q^2-m_H^2+F_5)(m_W^2+F_1+q^2F_2)}, \\
D^{\mu}_{H^+W^+} &= \frac{iq^\mu F_4}{q^2F_3F_4-(q^2-m_H^2+F_5)(m_W^2+F_1+q^2F_2)}, \\
D_H &= \frac{i}{q^2-m_H^2+F_5-\dfrac{q^2F_3F_4}{m_W^2+F_1+q^2F_2}}.
\end{aligned} \tag{8}
$$

The corresponding propagator matrix for $W^- - H^-$ is obtained by the replacement $q^\mu \to -q^\mu$, $F_3(q^2) \leftrightarrow F_4(q^2)$. The above derivation is analogous to the description of the $\gamma - Z$ system [8]. A graphical representation of Eqs. (7) and (8) is given in Figs. 1-3.

The function $D_W^{\mu\nu}$, representing the WW element of the propagator matrix, can be decomposed into transverse and longitudinal pieces:

$$
D_W^{\mu\nu} = i(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2})G_T + i\frac{q^\mu q^\nu}{q^2}G_L \tag{9}
$$

with

$$
\begin{aligned}
G_T &= \frac{1}{q^2-m_W^2-F_1}, \\
G_L &= \frac{1}{m_W^2+F_1+q^2F_2-\dfrac{q^2F_3F_4}{q^2-m_H^2+F_5}}.
\end{aligned} \tag{10}
$$

It will turn out that only the longitudinal part G_L contributes to the partial width asymmetry. If the term proportional to F_3F_4 is dropped, the function G_L coincides

with that in Refs. [1, 2]. We work initially with the full expression in Eq. (8), in order to obtain results that are also valid for $q^2 \simeq m_H^2$, a region that is physically accessible if $m_H < m_t - m_b$.

3 Difference of Partial Widths

3.1 Asymmetry in Lepton Channels

The amplitude of the decay $t \to b l^+ \nu_l$, including vacuum polarization effects in the $W - H$ propagator, is given by the sum of the four diagrams shown in Fig. 4, and has the form

$$
\begin{aligned}
M_l = \frac{ig^2}{8} \quad \Big\{ & \quad A_l \overline{u}_b \gamma^\mu (1-\gamma_5) u_t \overline{u}_\nu \gamma_\mu (1-\gamma_5) v_l \\
+ & \quad B_l \overline{u}_b (1+\gamma_5) u_t \overline{u}_\nu (1+\gamma_5) v_l + D_l \overline{u}_b (1-\gamma_5) u_t \overline{u}_\nu (1+\gamma_5) v_l \Big\}
\end{aligned} \qquad (11)
$$

with

$$
\begin{aligned}
A_l &= G_T, \\
B_l &= \frac{m_t m_l}{m_W^2} \Big\{ \frac{m_W^2}{q^2}(G_T + G_L) + Y^* Z G_5 + m_W N (Y^* F_4 + Z F_3) \Big\}, \\
D_l &= \frac{m_b m_l}{m_W^2} \Big\{ -\frac{m_W^2}{q^2}(G_T + G_L) + X^* Z G_5 + m_W N (X^* F_4 - Z F_3) \Big\},
\end{aligned} \qquad (12)
$$

where $N \equiv [(m_W^2 + F_1 + q^2 F_2)(q^2 - m_H^2 + F_5) - q^2 F_3 F_4]^{-1}$ and $G_5 \equiv N(m_W^2 + F_1 + q^2 F_2)$. The corresponding decay amplitude for $\overline{t} \to \overline{b} l^- \overline{\nu}_l$ is

$$
\begin{aligned}
\overline{M}_l = \frac{ig^2}{8} \quad \Big\{ & \quad \overline{A}_l \overline{v}_t \gamma^\mu (1-\gamma_5) v_b \overline{u}_l \gamma_\mu (1-\gamma_5) v_\nu \\
+ & \quad \overline{B}_l \overline{v}_t (1-\gamma_5) v_b \overline{u}_l (1-\gamma_5) v_\nu + \overline{D}_l \overline{v}_t (1+\gamma_5) v_b \overline{u}_l (1-\gamma_5) v_\nu \Big\}
\end{aligned} \qquad (13)
$$

with

$$
\overline{A}_l = G_T,
$$

$$\overline{B}_l = \frac{m_t m_l}{m_W^2}\Big\{\frac{m_W^2}{q^2}(G_T+G_L)+YZ^*G_5+m_W N(YF_3+Z^*F_4)\Big\},$$
$$\overline{D}_l = \frac{m_b m_l}{m_W^2}\Big\{-\frac{m_W^2}{q^2}(G_T+G_L)+XZ^*G_5+m_W N(XF_3-Z^*F_4)\Big\}. \quad (14)$$

The matrix elements M_l and $\overline{M}_l$ yield the following asymmetry between the partial widths:

$$\begin{aligned}
\Delta_{l\nu_l} &\equiv \Gamma(\overline{t}\to\overline{b}l^-\overline{\nu}_l)-\Gamma(t\to bl^+\nu_l)\\
&= \frac{1}{2m_t}\int\frac{d^3p_b}{(2\pi)^3 2E_b}\frac{d^3p_l}{(2\pi)^3 2E_l}\frac{d^3p_\nu}{(2\pi)^3 2E_\nu}(2\pi)^4\delta^{(4)}(p_t-p_b-p_l-p_\nu)\\
&\qquad\cdot\Big\{\overline{|\overline{M}_l|^2}-\overline{|M_l|^2}\Big\}\\
&= \frac{g^4}{2^{11}\pi^3 m_t^3}\int\limits_{m_l^2}^{(m_t-m_b)^2}\frac{dq^2}{q^4}\lambda(q^2,m_t^2,m_b^2)(q^2-m_l^2)^2\Big\{\\
&\quad (|\overline{B}_l|^2-|B_l|^2+|\overline{D}_l|^2-|D_l|^2)q^2(m_t^2+m_b^2-q^2)\\
&\quad +4Re(\overline{B}_l^*\overline{D}_l-B_l^*D_l)m_t m_b q^2\\
&\quad -2ReA^*(\overline{B}_l-B_l)m_t m_l(m_t^2-m_b^2-q^2)\\
&\quad -2ReA^*(\overline{D}_l-D_l)m_b m_l(m_t^2-m_b^2+q^2)\Big\}, \qquad (15)
\end{aligned}$$

where $\lambda(a,b,c)=(a^2+b^2+c^2-2ab-2ac-2bc)^{1/2}$. Substituting the expressions for $A_l, B_l, D_l, \overline{A}_l, \overline{B}_l, \overline{D}_l$ in the above integrand, we find (as anticipated) that terms proportional to the transverse propagator G_T cancel completely. Expressed in terms of the functions $F_i(q^2)$, the asymmetry involves only the quantities $Im(F_1+q^2F_2)(q^2)$, $(F_3-F_4^*)(q^2)$ and $ImF_5(q^2)$. Representing the self-energies by fermion loops, these terms are

$$\begin{aligned}
Im(F_1+q^2F_2)(q^2) &= \frac{g^2}{16\pi}\Big\{\frac{N_c\lambda(q^2,m_u^2,m_d^2)}{2q^4}\Theta[q^2-(m_u+m_d)^2]\\
&\quad\cdot(-m_u^4-m_d^4+q^2m_u^2+q^2m_d^2+2m_u^2m_d^2)\\
&\quad+(u,d)\to(c,s)+\sum_{l=e,\mu,\tau}\frac{(q^2-m_l^2)^2}{2q^4}m_l^2\Theta(q^2-m_l^2)\Big\},
\end{aligned}$$

$$
\begin{aligned}
(F_3-F_4^*)(q^2) &= \frac{g^2}{16\pi}\Big\{\frac{iN_c\lambda(q^2,m_u^2,m_d^2)}{q^4}\Theta[q^2-(m_u+m_d)^2] \\
&\quad [X^*\frac{m_d^2}{m_W}(q^2+m_u^2-m_d^2)-Y^*\frac{m_u^2}{m_W}(q^2-m_u^2+m_d^2)] \\
&\quad +(u,d)\rightarrow(c,s)+\sum_{l=e,\mu,\tau}\frac{i(q^2-m_l^2)^2}{q^4}(-Z^*)\frac{m_l^2}{m_W}\Theta(q^2-m_l^2)\Big\}, \\
ImF_5(q^2) &= \frac{g^2}{16\pi}\Big\{\frac{N_c\lambda(q^2,m_u^2,m_d^2)}{2q^2}[(|Y|^2\frac{m_u^2}{m_W^2}+|X|^2\frac{m_d^2}{m_W^2})(q^2-m_u^2-m_d^2) \\
&\quad -\frac{m_u^2m_d^2}{m_W^2}4ReXY^*]\Theta[q^2-(m_u+m_d)^2]+(u,d)\rightarrow(c,s) \\
&\quad +\sum_{l=e,\mu,\tau}\frac{(q^2-m_l^2)^2}{2q^2}|Z|^2\frac{m_l^2}{m_W^2}\Theta(q^2-m_l^2)\Big\}. \qquad (16)
\end{aligned}
$$

One finds that the contribution of the lepton loops (the pieces $\sum_{l=e,\mu,\tau}$) to the asymmetry vanishes identically, leaving as the final result

$$
\begin{aligned}
\Delta_{l\nu_l} &= \frac{g^6N_cm_l^2}{2^{14}\pi^4m_t^3m_W^6}\int_{Max[(m_c+m_s)^2,m_l^2]}^{(m_t-m_b)^2}\frac{dq^2}{q^6}\frac{\lambda(q^2,m_t^2,m_b^2)\lambda(q^2,m_c^2,m_s^2)(q^2-m_l^2)^2}{(q^2-m_H^2)^2+m_H^2\Gamma_H^2} \\
&\Big\{\Big[m_t^2m_s^2(m_t^2-m_b^2-q^2)(q^2+m_c^2-m_s^2) \\
&\qquad -m_b^2m_c^2(m_b^2-m_t^2-q^2)(q^2+m_s^2-m_c^2)\Big] \\
&\quad\cdot\Big[(1-\frac{m_H^2}{q^2})Im(XY^*-XZ^*-YZ^*) \\
&\qquad +|Y|^2ImXZ^*-|Z|^2ImXY^*+|X|^2ImYZ^*\Big] \\
&\quad +2ImYZ^*|X+Y|^2m_t^2m_c^2\Big[q^2(m_s^2-m_b^2)+m_b^2m_c^2-m_t^2m_s^2\Big] \\
&\quad +2ImXZ^*|X+Y|^2m_b^2m_s^2\Big[q^2(m_t^2-m_c^2)+m_b^2m_c^2-m_t^2m_s^2\Big]\Big\} \\
&+ (c,s)\rightarrow(u,d) \\
&\equiv \Delta_{l\nu_l}(c,s)+\Delta_{l\nu_l}(u,d). \qquad (17)
\end{aligned}
$$

We have introduced here the notation $\Delta_{l\nu_l}(c,s)$ to signify the contribution of the (c,s) loop to the asymmetry in the channel $l\nu_l$. Similarly, $\Delta_{l\nu_l}(u,d)$ denotes the contribution of the (u,d) loop. In deriving (17), we have used the optical theorem

in the form $ImF_5(q^2=m_H^2)=m_H\Gamma_H$, and have neglected terms of relative order g^2.

3.2 Asymmetry in Quark Channels

In complete analogy to the lepton case, the matrix elements for the decays $t\to bc\overline{s}$, $\overline{t}\to\overline{b}\overline{c}s$ are

$$
\begin{aligned}
M=\frac{ig^2}{8} & \Big\{ A\overline{u}_b\gamma^\mu(1-\gamma_5)u_t\overline{u}_c\gamma_\mu(1-\gamma_5)v_s \\
&+ B\overline{u}_b(1+\gamma_5)u_t\overline{u}_c(1+\gamma_5)v_s + C\overline{u}_b(1+\gamma_5)u_t\overline{u}_c(1-\gamma_5)v_s \\
&+ D\overline{u}_b(1-\gamma_5)u_t\overline{u}_c(1+\gamma_5)v_s + E\overline{u}_b(1-\gamma_5)u_t\overline{u}_c(1-\gamma_5)v_s\Big\}, \\
\overline{M}=\frac{ig^2}{8} & \Big\{ \overline{A}\overline{v}_t\gamma^\mu(1-\gamma_5)v_b\overline{u}_s\gamma_\mu(1-\gamma_5)v_c \\
&+ \overline{B}\overline{v}_t(1-\gamma_5)v_b\overline{u}_s(1-\gamma_5)v_c + \overline{C}\overline{v}_t(1-\gamma_5)v_b\overline{u}_s(1+\gamma_5)v_c \\
&+ \overline{D}\overline{v}_t(1+\gamma_5)v_b\overline{u}_s(1-\gamma_5)v_c + \overline{E}\overline{v}_t(1+\gamma_5)v_b\overline{u}_s(1+\gamma_5)v_c\Big\}, \qquad (18)
\end{aligned}
$$

where

$$
\begin{aligned}
A &= G_T, \\
B &= \frac{m_t m_s}{m_W^2}\Big\{\frac{m_W^2}{q^2}(G_T+G_L)-XY^*G_5+m_W N(Y^*F_4-XF_3)\Big\}, \\
C &= \frac{m_t m_c}{m_W^2}\Big\{-\frac{m_W^2}{q^2}(G_T+G_L)-|Y|^2G_5-m_W N(Y^*F_4+YF_3)\Big\}, \\
D &= \frac{m_b m_s}{m_W^2}\Big\{-\frac{m_W^2}{q^2}(G_T+G_L)-|X|^2G_5+m_W N(X^*F_4+XF_3)\Big\}, \\
E &= \frac{m_b m_c}{m_W^2}\Big\{\frac{m_W^2}{q^2}(G_T+G_L)-X^*YG_5+m_W N(YF_3-X^*F_4)\Big\} \qquad (19)
\end{aligned}
$$

and

$$
\begin{aligned}
\overline{A} &= A, \qquad \overline{C}=C, \qquad \overline{D}=D, \\
\overline{B} &= \frac{m_t m_s}{m_W^2}\Big\{\frac{m_W^2}{q^2}(G_T+G_L)-X^*YG_5+m_W N(YF_3-X^*F_4)\Big\},
\end{aligned}
$$

$$\overline{E} = \frac{m_b m_c}{m_W^2}\Big\{\frac{m_W^2}{q^2}(G_T+G_L) - XY^*G_5 + m_W N(Y^*F_4 - XF_3)\Big\}. \qquad (20)$$

Once again , the transverse propagator term G_T makes no contribution to the asymmetry, which is given by

$$\begin{aligned}\Delta_{cs} &\equiv \Gamma(\overline{t}\to\overline{b}\overline{c}s) - \Gamma(t\to bc\overline{s})\\ &= \frac{N_c g^4}{2^{11}\pi^3 m_t^3}\int\limits_{(m_c+m_s)^2}^{(m_t-m_b)^2}\frac{dq^2}{q^2}\lambda(q^2,m_t^2,m_b^2)\lambda(q^2,m_c^2,m_s^2)\Big\{\\ &\quad (|\overline{B}_0|^2 - |B_0|^2 + |\overline{E}_0|^2 - |E_0|^2)(m_t^2+m_b^2-q^2)(q^2-m_c^2-m_s^2)\\ &\quad -4Re[C_0^*(\overline{B}_0-B_0) + D_0^*(\overline{E}_0-E_0)]m_c m_s(m_t^2+m_b^2-q^2)\\ &\quad +4Re[D_0^*(\overline{B}_0-B_0) + C_0^*(\overline{E}_0-E_0)]m_t m_b(q^2-m_c^2-m_s^2)\Big\}, \qquad (21)\end{aligned}$$

where the subscript "0" means the expressions (19) and (20) without the terms proportional to G_T. Expressed in terms of the functions $F_i(q^2)$, the asymmetry involves only the combinations given in Eq. (16), yielding as the final result

$$\begin{aligned}\Delta_{cs} &= \frac{N_c^2 g^6}{2^{13}\pi^4 m_t^3 m_W^6} Im XY^*|X+Y|^2 \int\limits_{Max[(m_c+m_s)^2,(m_u+m_d)^2]}^{(m_t-m_b)^2}\frac{dq^2}{q^6}\\ &\quad \frac{\lambda(q^2,m_c^2,m_s^2)\lambda(q^2,m_u^2,m_d^2)\lambda(q^2,m_t^2,m_b^2)}{(q^2-m_H^2)^2+m_H^2\Gamma_H^2}\cdot f(q^2,m_t^2,m_b^2,m_c^2,m_s^2,m_u^2,m_d^2)\\ &- \sum_{l=e,\mu,\tau}\Delta_{l\nu_l}(c,s), \qquad (22)\end{aligned}$$

where the last term follows from the relation $\Delta_{cs}(l\nu_l) = -\Delta_{l\nu_l}(cs)$, which we have checked explicitly. The function f is defined by

$$\begin{aligned}f(q^2,t,b,c,s,u,d) &= q^4(tbcd - tbsu - tcsd + tsud + bcsu - bcud)\\ &+ q^2(t^2csd - t^2sud - tbc^2d - tbcd^2 + tbs^2u + tbsu^2\\ &\quad +tcsd^2 - ts^2ud - b^2csu + b^2cud + bc^2ud - bcsu^2)\\ &\quad +(ts-bc)(su-cd)(td-bu). \qquad (23)\end{aligned}$$

It has the remarkable property of being antisymmetric under any one of the following exchanges:

$$(u,d) \leftrightarrow (c,s) \qquad ; \qquad (u,d) \leftrightarrow (t,b) \qquad ; \qquad (c,s) \leftrightarrow (t,b). \tag{24}$$

As a consequence of this asymmetry, we immediately see that (i) the (c,s) loop does not contribute to Δ_{cs}, (ii) the analogous result for Δ_{ud} is obtained by interchanging (c,s) and (u,d) in Eq. (22), and (iii) the asymmetries in the various channels satisfy the relation

$$\Delta_{cs} + \Delta_{ud} + \Delta_{\tau\nu_\tau} + \Delta_{\mu\nu_\mu} + \Delta_{e\nu_e} = 0, \tag{25}$$

implying the equality of total width of t and $\bar{t}$, mandated by CPT invariance.

4 Comments

(i) Our results fulfil all the general constraints on partial width asymmetries noted by Wolfenstein [6]. In particular, the asymmetry in a channel f associated with a loop n satisfies

$$\Delta_f(n) = -\Delta_n(f) \tag{26}$$

and vanishes when $n = f$.

(ii) A characteristic feature of $W^+ - H^+$ interference is the result that the asymmetry $\Delta_q(q')$ in the quark channel q, arising from a quark loop q', is proportional to the function $f(q^2, m_t^2, m_b^2, m_1^2, m_2^2, m_3^2, m_4^2)$ defined in Eq. (23), where (m_1, m_2) and (m_3, m_4) are the masses of the quark doublets contained in q and q'. This implies that the specific asymmetry $\Delta_q(q')$ vanishes when one of the masses (m_1, m_2) and one of the masses (m_3, m_4) is zero. For a similar reason, the asymmetry in a lepton

channel l due to a lepton loop l' vanishes, even for $l \neq l'$, since the two doublets necessarily contain two massless neutrinos.

(iii) The fact that the asymmetries $\Delta_{l\nu_l}$, Δ_{cs} and Δ_{ud} given by Eqs. (17) and (22) satisfy the CPT condition (25) is a nontrivial test of the full $W^+ - H^+$ propagator constructed in Eq. (8). In particular, neglect of the off-diagonal terms F_3 and F_4 leads to conflict with CPT invariance. These terms have not been considered in previous work.

(iv) Our results for $\Delta_{\tau\nu_\tau}$ and Δ_{cs} do not coincide with those in Refs. [1, 2]. For instance, these earlier papers found an asymmetry $\Delta_{\tau\nu_\tau}$ proportional to $m_\tau^2 m_c^2$. By contrast, the leading term of our result (Eq. (17)) is proportional to $m_\tau^2 m_s^2$. We have been able to trace the difference to the neglect of the off-diagonal part of the $W^+ - H^+$ propagator in Refs. [1, 2], which inevitably leads to a violation of the CPT condition (Eq. (25)).

(v) In the absence of any scalar interaction of the form $\mathcal{L}_H$, the transverse and longitudinal parts of the propagator $D_W^{\mu\nu}$ obtained by us agree with those in Refs. [1, 2, 4].

(vi) Numerically, the partial width asymmetries resulting from $W^+ - H^+$ interference, in the models discussed here, are exceedingly small. As pointed out in Ref. [1], larger differences between $t \to b\tau^+\nu_\tau$ and $\bar{t} \to \bar{b}\tau^-\bar{\nu}_\tau$ occur if one compares the spectra of these reactions, not only in the variable q^2 but also in the complementary Dalitz variable $u = (p_\tau + p_b)^2$ [9, 10]. Likewise, larger asymmetries are possible if one compares the τ^+ and τ^- polarization [11]. Whereas the partial width asymmetry discussed in this paper involves only the longitudinal part of the W propagator, these alternative effects involve the transverse part, and do not necessarily require absorptive phases associated with final state interactions.

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Figure Captions

Fig. 1. Diagonal and non-diagonal one-particle-irreducible self-energies of the $W-H$ system (Eq. (7)).

Fig. 2. Graphical representation of the "pure" W and H propagators, neglecting $W-H$ mixing.

Fig. 3. Graphical representation of the full $W-H$ propagator (Eq. (8)), in terms of the "pure" W and H propagators defined in Fig. 2.

Fig. 4. Feynman diagrams contributing to the reaction $t \to b\tau^+\nu_\tau$.

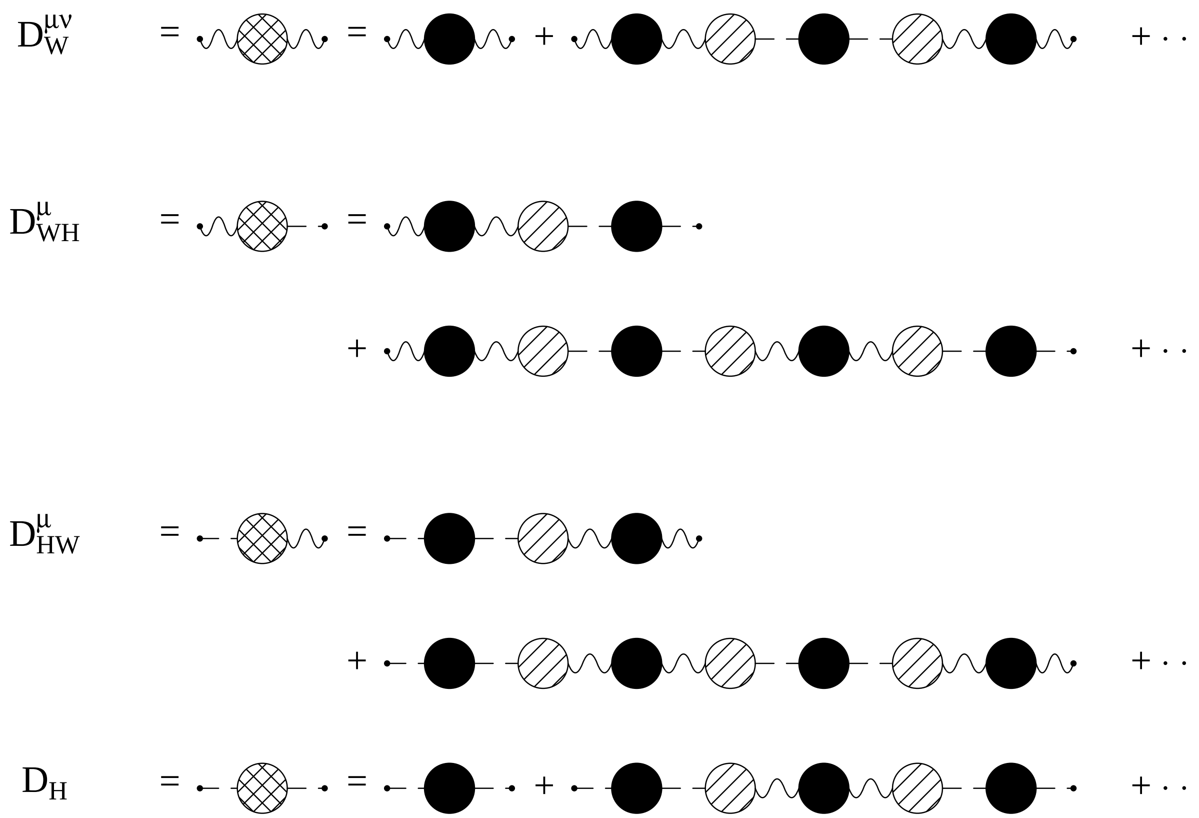

Fig. 3.

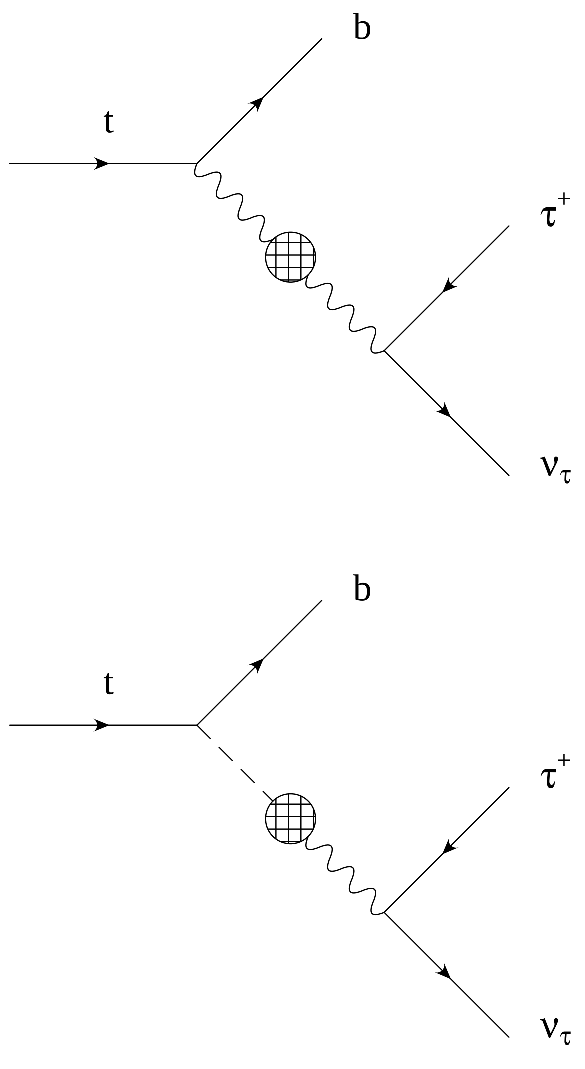

Fig. 4.

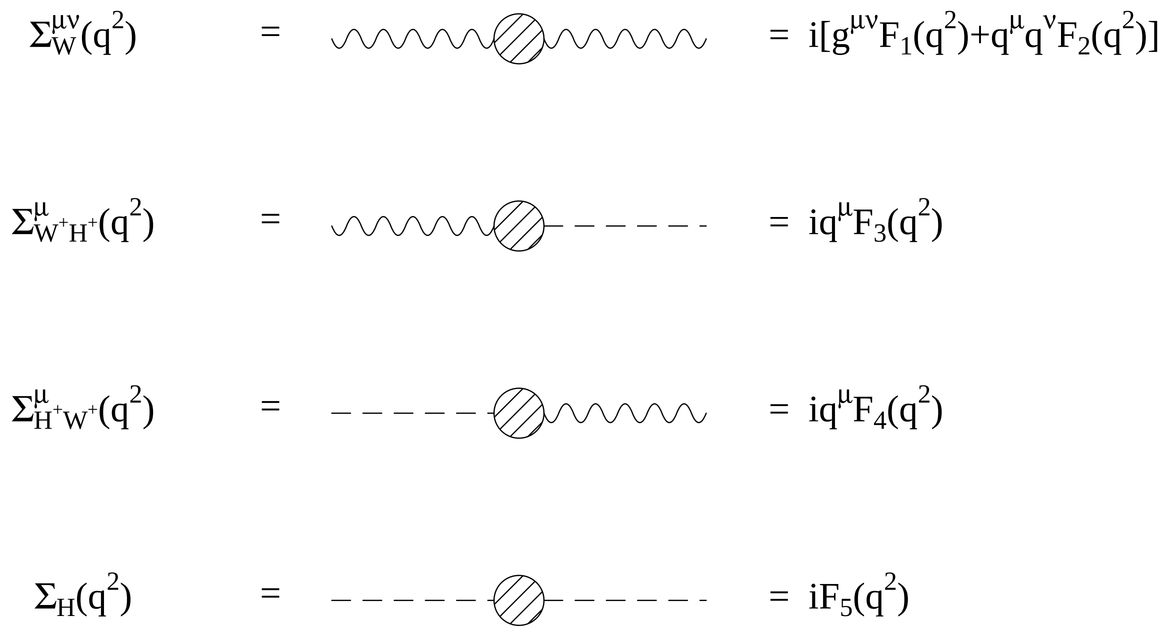
$\Sigma_W^{\mu\nu}(q^2)$ = = $i[g^{\mu\nu}F_1(q^2)+q^\mu q^\nu F_2(q^2)]$
$\Sigma^\mu_{W^+H^+}(q^2)$ = = $iq^\mu F_3(q^2)$
$\Sigma^\mu_{H^+W^+}(q^2)$ = = $iq^\mu F_4(q^2)$
$\Sigma_H(q^2)$ = = $iF_5(q^2)$

Fig. 1.

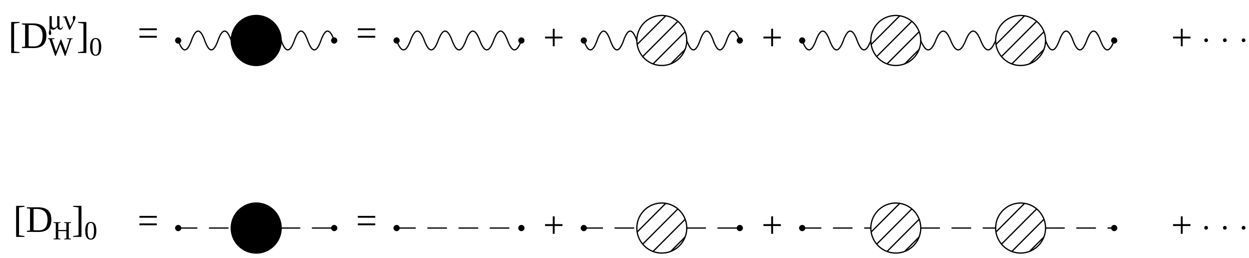
$[D_W^{\mu\nu}]_0$ = = + + $+ \cdots$
$[D_H]_0$ = = + + $+ \cdots$

Fig. 2.